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Original Article

Generalized modified Levakovic growth function for aquatic species

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Abstract: Growth is a concept that includes social, economic and physical sub-processes and it also shows itself in the field of fisheries. It is extremely important to obtain the appropriate mathematical model for growth for aquatic species. The growth modelling for the aquatic species contains the phenomena of bio-diversity, formation and population dynamics of aquatic species or stock estimation for the creature. And this study aims to design and implement a new mathematical model for growth process for aquatic species as Generalized Modified Levakovic Growth (GLM) model.

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Introduction

Mathematical growth models are used to predict the growth of biological systems and intraspecific population dynamics. Growth functions are simple, nonlinear mathematical equations that describe the change in the size of an individual or population over time. In some cases, simple exponential growth functions can model the growth. Growth curves serve to express the development of growth parameters over time mathematically. It is unthinkable to show an ever-increasing trend for the concept of growth; because nothing can be expected to increase continuously (Bertalanffy, 1938). In the simplest terms, even the population growth has a certain saturation level, so called "*carrying capacity*". This value assumes a numerical upper bound role for growth size.

Von Bertalanffy (1938) proposed the first important research on the mathematical expressions of the size of the organism. In addition, the Richards model (Richards, 1959), obtained by the modification of the von Bertalanffy model, has found a wide range of application. Lifeng et al. (1998) was implemented Richards model to various species such as *Amoeba proteus*, rice, fish, cattle and deer. Later, Brich (1999) defined a new generalized logistic model and compared it to Richards model. To compare and fit the models, Brich (1999) applied the real data set which was also used in the Richards study. The use of the growth models is not only limited to the field of biology. They can be applied to many other fields such as determining the market volume in economics (Fisher and Fry, 1971; Fingleton and López-Bazo, 2006).

Levakovic (1935) model, one of the functions used in the modeling of growth, is actually the generalized form of the Hossfeld (Woollons et al, 1990) model (Lee, 2000). Since the Levakovic model is not very useful in practice, this function is generally used unified with other growth models (Bontemps and Duplat, 2012). The model known as Levakovic I is known as the best model among 4-parameter models (Zeide, 1993). Based on above-mentioned background, in the first part of this study, construction of a new growth model is defined by inspiring Levakovic function called as Generalized Modified Levakovic Growth (GLM) model to apply for aquatics. In the second section, the prominent features of GLM model is discussed and finally conclusion section is given.

(1) Construction of generalized modified Levakovic growth function

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Statistical modeling with growth data creates a more efficient way to understand biological progress. In this section, a new growth model is constructed as Generalized Modified Levakovic (GLM) growth model.

The relative growth rate: The relative growth rate is calculated as: $z(t) = \frac{\frac{dy(t)}{dt}}{y(t)} = \frac{\mathrm{d}}{\mathrm{dt}}\ln t$, $z(t) > 0$, and we define a new relative rate for growth as follows:

$$z(t) = \mathrm{c\ a\ b}\, e^{-at}\, (1 - b\, e^{-at})^{-1} \ldots\ldots\ldots\ldots\ldots (1)$$

Where $y_{\infty} = \lim_{t\to\infty} y(t)$ and a, b, c are any real numbers with a, b, c ≥ 0, *t* is time variable, $t \geq 0$ and $y(t)$ is current size at time t. Thus:

$$\frac{\frac{dy(t)}{dt}}{y(t)} = \mathrm{c\ a\ b}\, e^{-at}\, (1 - b\, e^{-at})^{-1} \ldots\ldots\ldots\ldots (2)$$

From now on, it is easy to define the instant rate $y(t)$. To obtain the $y(t)$ function which gives the current size at time t, the integration of the Equation 2 should be find.

$$\int \frac{\frac{dy(t)}{dt}}{y(t)}\, dt = \int \mathrm{c\ a\ b}\, e^{-at}\, (1 - b\, e^{-at})^{-1}\, dt \ldots (3)$$

By using the change of variable technique, the integral in the Equation 3 can be evaluated as:

$$u = 1 - b\, e^{-at} \rightarrow du = a\, b\, e^{-at}\, dt$$

$$\ln y(t) = ln(1 - b\, e^{-at})^{c} + ln\, c_1$$

Where c_1 is an integration constant. Therefore, the formula for GLM growth function is:

$$y(t) = c_1\, (1 - b\, e^{-at})^{c} \ldots\ldots\ldots\ldots\ldots (4)$$

To see the c_1, let us analyze this equation as t tends to infinity: $Y_{\infty} = \lim_{t\to\infty} y(t) = c_1$ is an asymptote, i.e. this number gives the *carrying capacity*. Since $c_1 = Y_{\infty}$, the Equation 4 should be arranged as in Equation 5. Then, the general formula for GLM growth function would be:

$$y(t) = Y_{\infty}\, (1 - b\, e^{-at})^{c} \ldots\ldots\ldots\ldots\ldots (5)$$

Growth rate: Using Equations 2 and 5, we define growth rate for the GLM growth function as follows:

$$\frac{dy(t)}{dt} = y_{\infty} \mathrm{c\ a\ b}\, e^{-at} (1 - b\, e^{-at})^{c-1} \ldots (6)$$

Where $y_{\infty} = \lim_{t\to\infty} y(t)$, a, b, c are any real numbers with a, b, c, t≥0 , where t is time variable and $y(t)$ is current size at time t.

(2) Properties of generalized modified Levakovic growth function

To obtain some identifier features of the GLM model, the initial value of GLM model is:

$$y(0) = Y_{\infty}\, (1 - b\,)^{c}$$

Initial behavior of the growth rate can be reached by taking zero in the first derivative of GLM model that is:

$$y'(t) = \left.\frac{dy(t)}{dt}\right]_{t=0} = y_{\infty} \mathrm{c\ a\ b}\, (1 - b\,)^{c-1}$$

as the initial behavior of the GLM function. Another important feature for a growth curve is the critical points that can be obtained by reaching the points where the first derivative is equal to zero. Therefore, we should take *dy(t)/dt* = 0.

$$\frac{dy(t)}{dt} = y_{\infty} \mathrm{c\ a\ b}\, e^{-at} (1 - b\, e^{-at})^{c-1} = 0$$

By exponential transformation, $t_{critical}$=ln $b^{1/a}$ can be calculated. Then, the value of the GLM growth model at critical point would be $y(t_{critical}) = 0$.

Inflection point: Inflection points are the points where the graph of a function changes curvature i.e. the point where the curve turns to convex from concave or vice versa. Therefore, it is one of the most important properties that gives information about the structure of a curve. Finding these points is possible by setting the second derivative to zero and often requires a rather tedious calculation process. In this section, the inflection point of the GLM function is calculated.

$$\frac{d^2y(t)}{dt^2} = -y_{\infty} \mathrm{c}\, a^2\, \mathrm{b}\, e^{-at} (1 - b\, e^{-at})^{c-1} [1 + b\, (1 - c) e^{-at} (1 - b\, e^{-at})^{-1}] \ldots (7)$$

For $t_1 = ln(bc)^{1/a}$ or $t_2 = \ln b^{1/a}$ second derivative given in Equation 7 is equal to 0, i.e. $\frac{d^2y(t)}{dt^2}$=0 for t_1 and t_2.

Population size at inflection point: Notice that $t_2 = \ln b^{1/a}$ is the critical value for GLM growth function, therefore by consideration of $t_1 = ln(bc)^{1/a}$, we can calculate y(t) at inflection point $t_1 = ln(bc)^{1/a}$ as follows:

$$\mathrm{y}(t_1) = Y_{\infty}\ (1 - \frac{1}{c})^{c}$$

Which gives the population size at point of inflection. And population at inflection point is:

$$\mathrm{y}(t_1) = Y_{\infty}\ (1 - \frac{1}{c})^{c} \ldots\ldots\ldots\ldots\ldots (8)$$

The maximum specific growth rate: Using the value of $t_1 = ln(bc)^{1/a}$ in the first derivative of $y(t)$, that is calculating the following equation,

$$y'(t_1) = Y_\infty\, a \left(1 - \frac{1}{c}\right)^{c-1} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

The slope of the curve at inflection point $t_1 = ln(bc)^{1/a}$ can be obtained.

Lag Time: The points where the growth reaches its maximum and stops the lag time, denoted by λ. This point is the x-intercept of the tangent curve at the inflection point. Again, by considering $t_1 = ln(bc)^{1/a}$ as an inflection point, the lag time λ can be reached by writing tangent equation about (t_1, y(t_1)) point. Then using the Equations 8 and 9, the tangent equation would be as:

$$y - y(t_{inf}) = y'(t_{inf}).\ (t - t_{inf})$$

At (λ, 0), this equation gives the lag time as:

$$0 - y(t_1) = y'(t_1).\ (\lambda - t_1)$$

$$-Y_\infty\ (1 - \frac{1}{c})^{c} = Y_\infty\, a\, (1 - \frac{1}{c})^{c-1}\, (\lambda - ln(bc)^{1/a})$$

And by performing some manipulations,

$$\lambda - \ln(bc)^{\frac{1}{a}} = \frac{c-1}{c\, a}$$

equation can be reached. Then the lag time is as follows:

$$\lambda = \frac{c - 1 + \ln(bc)^{c}}{c\, a}$$

Conclusion

The time-dependent evolution/variation seen in a population or living thing is expressed by growth functions. There is an enormous amount of empirical and theoretical study relating to the growth. Because, as emphasized in this study, the importance of accurate modeling of growth is known by scientists. The model presented in the study will contribute to study of biological systems particularly aquatics. In addition, the application of this model to fisheries data sets in future studies will provide new expansions and will lead to progress in growth estimation.

References

Birch C.P. (1999). A new generalized logistic sigmoid growth equation compared with the Richards growth equation. Annals of Botany, 83(6): 713-723.

Bontemps J.D., Duplat P. (2012). A non-asymptotic sigmoid growth curve for top height growth in forest stands. Forestry: An International Journal of Forest Research, 85(3): 353-368.

Fingleton B., López-Bazo E. (2006). Empirical growth models with spatial effects. Papers in Regional Science, 85(2): 177-198.

Fisher T.C., Fry R.H. (1971). Tech. Forecasting Soc. Changes, 3: 75.

Lee A. (2000). The predictive ability of seven sigmoid curves used in modelling forestry growth: a thesis submitted to the Institute of Information Sciences and Technology in partial fulfilment of the requirements for the degree of Master of Applied Statistics at Massey University, February, 2000 (Doctoral dissertation, Massey University).

Levakovic A. (1935) Analytical form of growth laws. Glasnik za Sumske Pokuse (Zagreb), 4: 189-282.

Lifeng X., Minggao S., Yuanjun W. (1998). Richards growth model of living-organism. Journal of Biomathematics, 13(3): 348-353.

Richards F.J. (1959). A flexible growth function for empirical use. Journal of experimental Botany, 10(2): 290-301.

Von Bertalanffy L. (1938). A quantitative theory of organic growth (inquiries on growth laws. II). Human Biology, 10(2): 181-213.

Woollons R.C., Whyte A.G.D., Xu L. (1990). The Hossfeld function: An alternative model for depicting stand growth and yield. Journal of the Japanese Forest, 15: 25-35.

Zeide B. (1993). Analysis of growth equations. Forest Science, 39(3): 594-616.